MATH-911 Special Functions

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: This course deals with the theory of functions of real and complex variables. While the original definition of a function may be in a more limited domain it can often be extended to larger domains by analytic continuation. As such, integral transforms that extend the domain of applicability are needed to study the functions in themselves. We will first discuss integral transforms and "fractional calculus" and go on to the special functions used in other areas of mathematics, in Statistics and in number. We then go on to the special functions of mathematical physics that originated as solutions of 2nd order linear ordinary differential equations and their continuation by integral representations.

Core Contents: Transform Methods, Fractional Calculus, Special Functions.

Detailed Course Contents: The integral operator and integral transforms. Linear and non- linear integral transforms. Fourier transforms of classical functions and conditions for existence. Properties of Fourier transform. Convolutions properties of Fourier transform. Distributions and generalized functions. Fourier transforms of generalized functions. Poisson summation formulae and applications. The Laplace transform and conditions for its existence. Basic properties of Laplace transform. Convolutions. Inverse Laplace transforms. Differentiation and integration of Laplace transforms. Use of Laplace transforms for differential and integral equations. Fractional calculus and its applications. Fractional differential and integral equations. The Hilbert transform and its properties. Extension to the complex domain. The Steiltjes transform its properties and inversion theorems. The Mellin transform. The gamma and beta functions and their integral representations. Properties and asymptotic expansion of the gamma function. The probability integral and its properties for real and complex domains. The exponential and logarithmic integrals. Hypergeometric functions and Legendre functions. The hypergeometric series and its analytic continuation. Properties of the hypergeometric functions. Confluent hypergeometric functions. Generalizedhypergeometric functions.

Learning Outcomes: On successful completion of this course, students will be able to:

- Understand the concepts of integral transforms.
- Understand the notion of fractional calculus.
- Know the transform methods and special functions with their properties and applications.

Text books:

1. L. Debnath and D. Bhatta, Integral Transforms and Their Applications Chapman &Hall/CRC; Second Edition (October 2006)
2. N.N. Lebedev, Special Functions and their applications (tr. R.R. Silverman) DoverPublications (Revised Editions, June 1972)

Reference Books:

1. M. Ya. Antimirov, A. A. Kolyshkin and Remi Vaillancourt, Applied Integral Transforms, The American Math. Society, (1993)
2. Nikiforov and Uvarov, Special Functions of Mathematical Physics, Springer, 1988

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	2.1-2.5, 2.9	Fourier transforms of classical functions and conditions for existence. Properties of Fourier transform. Convolutions properties of Fourier transform.
2	2.10-2.13, 3.1-3.4	Fourier transforms of generalized functions. Poisson summation formulae and applications to the solution of differential and integral equations. The Laplace transform and conditions for its existence. Basic properties of the Laplace transform.
3	3.4 – 3.7	Convolutions, Inverse Laplace transforms. Differentiation and integration of Laplace transforms.
4	5.1 – 8.4	Fractional calculus and its applications. Fractional differential and integral equations
5	6.1-6.3	Laplace transform of fractional integrals and derivatives, Mittage-Lefller function and its properties, Fractional ordinary differential equations.
6	6.4,6.5	Fractional integral equations, Initial value problems for fractional differential equations
7	8.1-8.4	Mellin Transforms: Properties and application of Mellin transforms
8	8.5-8.7	Mellin transform of fractional integrals and derivatives
9	**Mid Semester Exam**	
10	9.1-9.4	The Hilbert transform and its properties, Extension to the complex domain
11	9.7-9.8	The Steiltjes transform its properties and inversion theorems.
12	NNL 1.1 – 1.5	The gamma and beta functions and their integral representations. Properties and asymptotic expansion of the gamma function. Incomplete gamma function.
13	2.1 - 2.4	The probability integral and its properties for real and complex domains. Asymptotic representation of probability integrals.
14	3.1 - 3.4	The exponential and logarithmic integrals. Asymptotic representation of exponential integrals.
15	7.1 – 7.6	Hypergeometric functions and Legendre functions. The hypergeometric series and its analytic continuation

16	9.1 – 9.5 9.7, 9.8	Properties of the hypergeometric functions. Confluent hypergeometric functions. Generalized hypergeometric functions
17		Review
18	**End semester exam**	